Two-Step Factor Models

Macroeconomic Forecasting with Principal Component Analysis

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Introduction & Motivation

Dataset & Targets

- Quarterly panel: 144 periods, $N = 97$ variables (date excluded)
- Time span: 1992–Q1 to 2027–Q4 (we train on realized data only)
- COVID era (2020 onward) treated as a structural break

Targets (Y): real GDP growth (transformed `rvgdp`), CHF/EUR exchange rate (`wkfreuro`), CPI inflation.

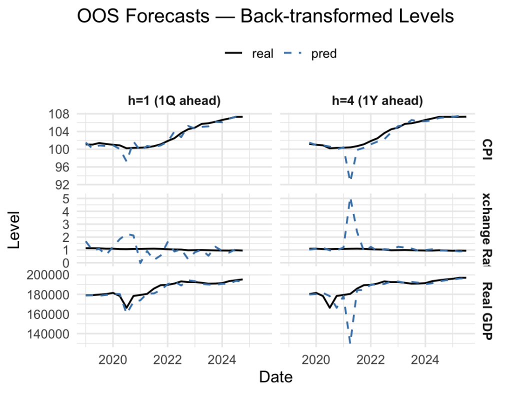
Out-of-Sample Around COVID-19 (h=1 and h=4)

Setup

- Now: Training stops pre-COVID (2019–Q4).
- OOS Forecast from previous models support this decision
- Horizons: **h=1** (1 quarter ahead) and **h=4** (1 year ahead).

Takeaway

- COVID is a large, unanticipated shock.
- Deviation visible *1 quarter* and *4 quarters* after the break.
- This period is inherently hard to model.



Two-Step Approach: Intuition

- **Step 1 (Dimension reduction):** map $N = 97$ variables to a few factors that capture common variation.
- **Step 2 (Prediction):** use factors ($+$ selected originals) in regressions for GDP growth, exchange rate, and inflation.

Goal: retain explainability while avoiding overfitting and multicollinearity.

Data Processing

Preparation & Transformations

- Normalize metadata
- Map units $\rightarrow$ transformations to induce stationarity:
  - `logdiff`: real/nominal quantities, indices $\Rightarrow$ quarterly growth rates
  - `diff`: rates, percentages, levels with stable mean (excluding exchange rate)
- Drop all-missing and near-zero-variance series.

Standardization & Sample

- Standardize transformed series (zero mean, unit variance).
- Predictors X: all stationary series excluding the target.
- Split sample: 80% initial training window; 20% pseudo-out-of-sample evaluation.

Principal Component Analysis

PCA: Intuition

- PCA finds orthogonal linear combinations explaining maximal variance.
- Reduces noise and multicollinearity in high dimensions.
- Provides low-dimensional summaries of co-movements (e.g., cycle/price/financial factors).

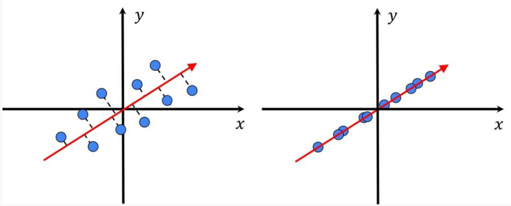


Figure 1: Illustration of variance-maximizing projection.

PCA: Formulation

$$X \in \mathbb{R}^{T \times N}, \quad X \approx F\Lambda^\top + E$$

$$\min_{F,\Lambda} \|X - F\Lambda^\top\|_F^2 \quad \text{s.t.} \quad \frac{1}{T}F^\top F = I_r$$

- F: factor scores ($T \times r$), Λ: loadings ($N \times r$)
- r: number of factors; E: residuals
- Solution via SVD on standardized X

Minimization of squared residuals, ensuring factors are orthogonal and have unit variance

PCA in R (minimal)

```r
bn_select_r_icp2 <- function(X,rmax=50){ # ICp2 over r; return r
  # ...implementation omitted for brevity...
}
X_imp <- X0; X_imp[!is.finite(X_imp)] <- NA
for(j in seq_len(ncol(X_imp))) X_imp[is.na(X_imp[,j]),j] <- mean(X_imp[,j],na.rm=TRUE)
r <- bn_select_r_icp2(X_imp, rmax=50)$r
pca <- prcomp(X_imp, center=TRUE, scale.=TRUE)  # stats::prcomp
eig <- pca$sdev^2; cumv <- cumsum(eig/sum(eig))
cat(sprintf("Bai Ng r=%d; CumVar[r]=%.2f\n", r, cumv[r]))
```

PCA for Macro Panels

- Starting point: $N = 94$ variables $\rightarrow$ low-dimensional $r \ll N$
- Benefits: reduce overfitting; capture latent drivers (cycle, prices, financial conditions)

Choosing the Number of Factors

- Caveat: correlated variables can inflate early eigenvalues if not standardized/filtered
- Bai–Ng information criteria select r (initially $r = 17$).
- Post-filter: if pairwise factor correlations > 0.9, keep one (parsimony).
- Cross-check with cumulative variance explained (elbow).

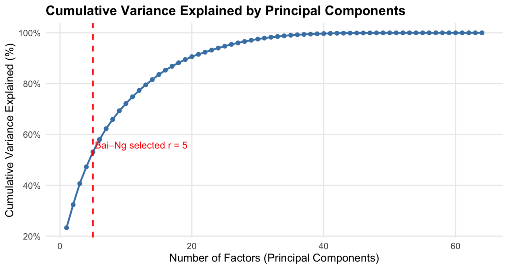


Figure 2: Cumulative variance explained vs. r.

VAR with PCA Factors

VAR: Idea & Specification

Goal: To capture the dynamic relationships among multiple macroeconomic variables or latent factors over time.

A VAR(p) model with K variables can be written as:

$$Y_t = c + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \varepsilon_t$$

where:
- Y_t is a $K \times 1$vector of variables (or factors) at time t;
- A_i are the coefficient matrices capturing lagged effects;
- ε_t is a vector of shocks (innovations) uncorrelated over time.

Economic Intuition
- Each variable depends on its own past values and on the past of other variables in the system.
- Captures **feedback loops** and **dynamic interdependencies**
- No structural or causal assumptions are imposed, it is a **data-driven** framework.

Economic Intuition and Dynamics

Feedback Mechanisms in the Swiss Economy

Real GDP Growth ↔ Inflation
Stronger economic growth increases demand for goods and labor, pushing prices upward. Rising inflation, if persistent, may prompt monetary tightening, which subsequently dampens growth.

Inflation ↔ Exchange Rate
Higher inflation erodes competitiveness, potentially weakening the currency. Conversely, a weaker exchange rate increases import prices, contributing to domestic inflation.

Exchange Rate ↔ GDP Growth
A stronger franc makes exports more expensive, reducing export demand and slowing growth. Economic weakness may reduce interest rates, weakening the currency.

Lag Structure and Dynamic Adjustment

The VAR model captures how shocks propagate through the economy over multiple quarters. Lags allow the model to represent realistic adjustment speeds: some variables respond immediately (e.g., asset prices), while others adjust gradually (e.g., employment, inflation expectations).

In our Model

In the **Dynamic Factor Model (DFM)** framework:

The **PCA** extracts a few common factors $F_t$that summarize hundreds of macro indicators.
The **VAR on factors** captures the **temporal evolution** of these latent factors:

$$F_t = c + A_1 F_{t-1} + A_2 F_{t-2} + \nu_t$$

From this process we generate forecasts for future factors $\hat{F}_{t+1}, \hat{F}_{t+2}, \hat{F}_{t+3}, \hat{F}_{t+4}$
which are then **mapped onto key target variables** (e.g., GDP, CPI).

$$\hat{F}_{t+1} = c + \hat{A} F_t + \hat{A} F_{t-1} + v_t$$

Mapping Factors to Target Variables

Now we connect these factors to our three target variables:

Target Variable Regression

$$\hat{Y}_{t+1} = \beta_{i,0} + \beta_{i,1}F_t + \beta_{i,2}F_{t-1} + \varepsilon_{i,t}$$

Where $Y_{i,t}$ is the target variable at time t
F_t = current factor(s), F_{t-1} = lagged factor(s). Estimated separately for each variable j

Now we have everything: Coefficients from the regression and future factors from the VAR
→ Use everything to predict $\hat{Y}_{i,t+1}$

$$\hat{Y}_{t+1} = \hat{\beta}_0 + \hat{\beta}_1\hat{F}_{t+1} + \hat{\beta}_2 F_t$$

$$\hat{Y}_{t+4} = \hat{\beta}_0 + \hat{\beta}_1\hat{F}_{t+4} + \hat{\beta}_2 \hat{F}_{t+3}$$

The Forecasting Chain: The factors act as **intermediaries** between the data and our target variables.

Expanding Window Evaluation

Out of Sample - Expanding Window

Objective: Simulate real-time forecasting conditions and evaluate the model's **out-of-sample (OOS)** predictive power.

An **expanding window** approach estimates the model recursively, increasing the training sample each period:

At each time t: *Estimate model on* $(1, \ldots, t) \rightarrow$ *Forecast at* $(t, +1, t + h)$

After each forecast, one new observation is added and the model is re-estimated.

<u>Economic Intuition:</u>

- Allows analysis of **forecast stability** over time.
- Mimics how central banks and forecasters update models with newly released data.
- Provides a realistic evaluation of model performance under real-time information constraints.

Mathematical Framework

Expanding Window Algorithm

For t = t_0, t_0+1, ..., T-h:

1. Estimate model on data {1, 2, ..., t}
2. Generate forecast $\hat{Y}_{\{t+h|t\}}$ using estimated model
3. Observe actual value Y_{t+h}
4. Compute forecast error $e_{i,t}$ = Y_{t+h} - $\hat{Y}_{\{t+h|t\}}$
5. Store results and proceed to next iteration

Notation and Definitions

t_0: Initial training period (e.g., 80% of data)
T: Total number of observations available
h: Forecast horizon (e.g., h=1 for 1-step, h=4 for 4-step ahead)
$\hat{Y}_{\{t+h|t\}}$ **:** Forecast of y_{t+h} made at time t
OOS origins: {t_0, t_0+1, ..., T-h} (number of origins = T- t_0-h + 1)

Key Property: Training set grows from {1,...,t_0} to {1,...,T-h}, ensuring each forecast is truly out-of-sample and model parameters evolve as new information arrives.

When to Use Each Method

Expanding Window: Preferred for macroeconomic forecasting where longer history improves estimates and structural change is gradual.

Rolling Window: Useful when recent data is more relevant (e.g., high-frequency trading) or when structural breaks are suspected.

Fixed Window: Rarely recommended for time-series forecasting; useful only when data is limited or for simple benchmarking exercises.

Forecast Evaluation Metrics: RMSE and MAE

To evaluate forecast accuracy, we need metrics that quantify the difference between predicted and actual values. We use two standard metrics: **RMSE (Root Mean Squared Error)** and **MAE (Mean Absolute Error)**.

Root Mean Squared Error (RMSE)

$$\text{RMSE} = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2$$

RMSE measures the **average magnitude of forecast errors**, giving more weight to larger errors due to squaring.

Mean Absolute Error (MAE)

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t|$$

MAE measures the **average absolute forecast error**, treating all errors equally regardless of direction. It is more robust to outliers than RMSE and easier to interpret: it represents the average forecast error in original units.

OOS Performance: DFM Model Across All Targets

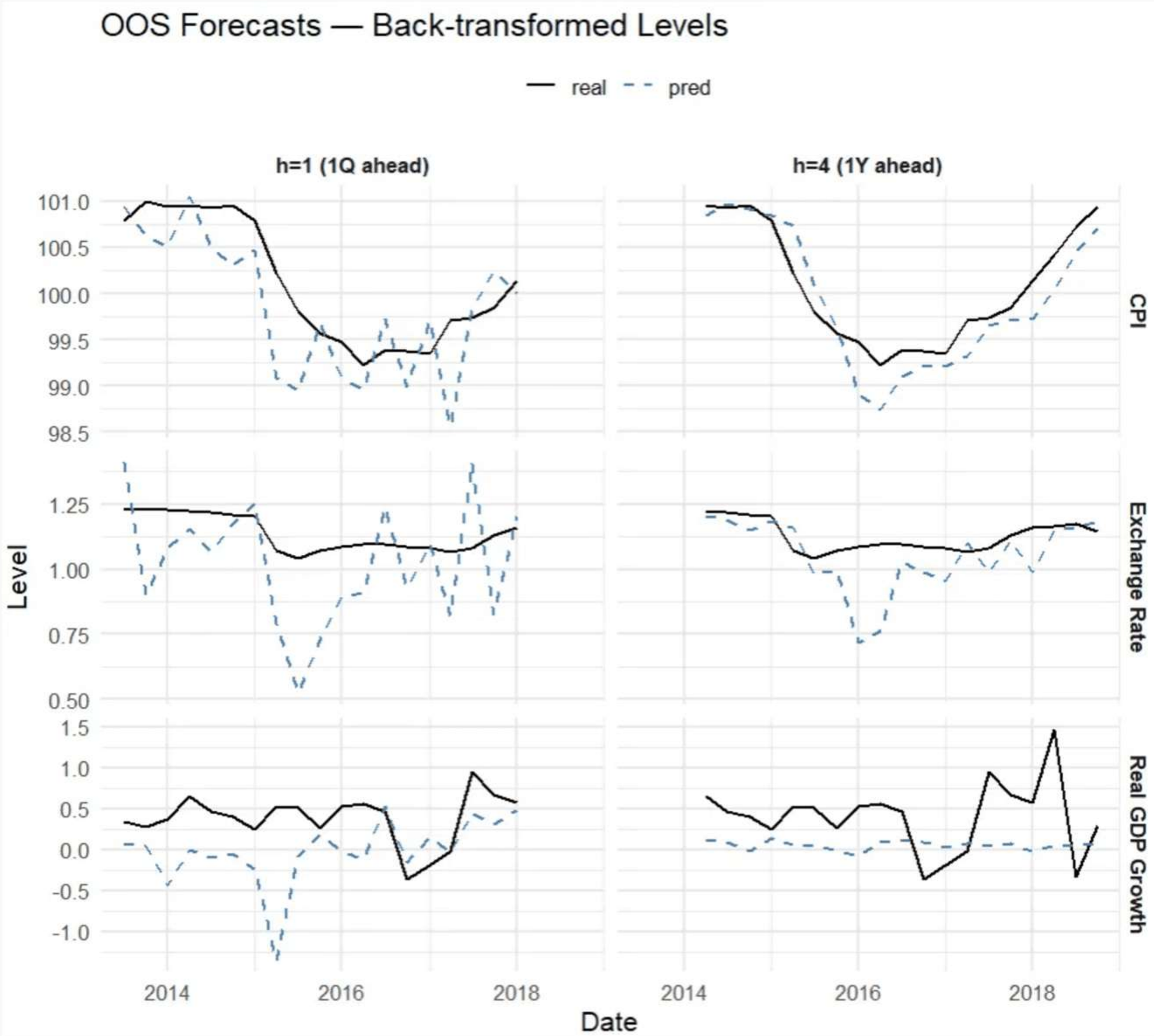
We evaluate the VAR model's ability to forecast **the targets** over the out-of-sample period (Q1 2013 – Q4 2019).

Variable	Metric	h=1 (1Q ahead)	h=4 (1Y ahead)
Real GDP Growth			
	RMSE	0.613	0.557
	MAE	0.466	0.473
Inflation (CPI)			
	RMSE	0.524	0.294
	MAE	0.425	0.245
Exchange Rate (CHF/EUR)			
	RMSE	0.233	0.135
	MAE	0.195	0.091

Key Insights

- GDP is the most volatile variable $\rightarrow$ harder to predict at short horizons.
- CPI errors decrease substantially at h=4 $\rightarrow$ model capture long-run inflation pressures.
- Exchange rate is more stable over time and less exposed to domestic shocks
- h=4 forecasts do not deteriorate $\rightarrow$ factors capture the **underlying business cycle**.

Expanding Window Learning Path



Nowcasting Framework

Nowcasting: Mathematical Framework

OOS (Expanding Window)

- **Goal:** Evaluate predictive power as if real-time
- **Data Cutoff:** Training data artificially restricted (e.g., Q4 2019)
- **Purpose:** Simulate historical forecasting accuracy

Nowcasting

- **Goal:** Produce best possible forecast for next quarters
- **Data Cutoff:** All available data up to Q2 2025
- **Purpose:** Generate current, most accurate forecast

Key Benefit: Incorporating the most recent information (including the post-COVID period) allows for more accurate estimation of the current economic state and factor dynamics.

Mathematical Approach

The same DFM structure (PCA for factors, VAR(2) for factor dynamics, OLS for targets) is applied.

1. Factor Extraction: Ft = PCA($X_{1:t}$)

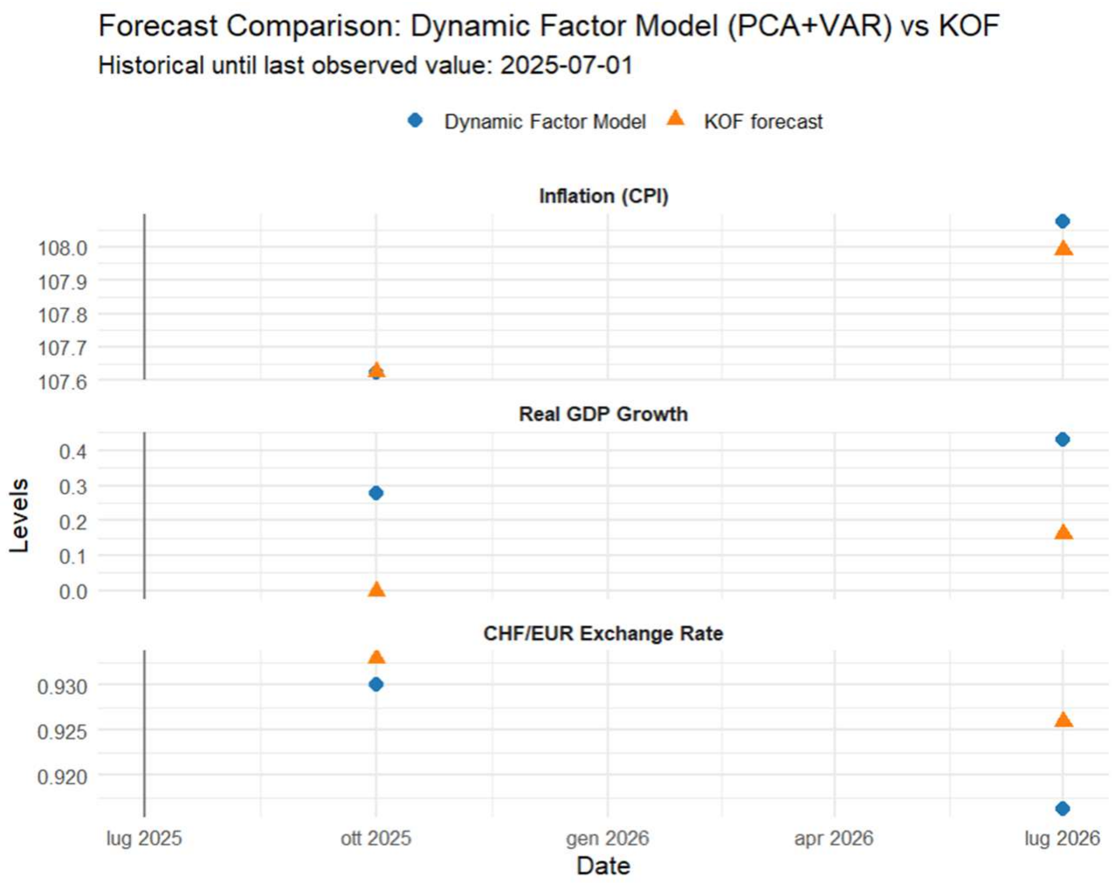
2. Factor Dynamics: Φ* = VAR(2) on $F_{1:t}$

3. Forecast: $\hat{Y}_{t+1} = \hat{\beta}_0 + \hat{\beta}_1 \hat{F}_{t+1} + + \hat{\beta}_2 F_t$

4. Base: Forecast based on most recent observed levels L_{now}

Nowcasting Results: DFM vs KOF Forecasts

Variable	DFM (Q4 2025)	KOF (Q4 2025)	DFM (Q3 2026)	KOF (Q3 2026)
Real GDP Growth (rvgdp)				
	0.28%	0.01%	0.43%	0.18%
Inflation (CPI)				
	107.62	107.62	108.07	108.05
Exchange Rate (wkfreuro)				
	0.930	0.929	0.916	0.925



FAVAR with PCA Factors

FAVAR: Mathematical Framework

In the **Factor-Augmented VAR (FAVAR)** framework, we combine the target variables and the latent factors into a single system, which allows for bidirectional feedback

The Combined Vector Z_t: $Z_t = [Y_t, F_t]$

Where:
- Y_t is the vector of our three target variables (Real GDP Growth, CPI, Exchange Rate)
- F_t is the vector of the common factors extracted via PCA

The VAR Specification

The dynamics of this combined system are modeled using a Vector Autoregression of order *p=2*:

$$Z_t = c + A_1 Z_{t-1} + A_2 Z_{t-2} + \varepsilon_t$$

Where:
- A_i are the coefficient matrices capturing lagged effects
- ε_t is the vector of innovations

DFM: VAR applied only to factors (F_t). Targets mapped separately via OLS.
FAVAR: VAR applied to the **entire system** (Z_t). Dynamics of targets and factors estimated **simultaneously**.

FAVAR: The Forecasting Chain

The Integrated Forecasting Step

Once the FAVAR model is estimated on the combined vector Z_t, the forecast for the entire system is generated directly from the VAR:

$$\hat{Z}_{t+h} = \hat{c} + \hat{A}_1\,\hat{Z}_{t+h-1} + \hat{A}_2\,\hat{Z}_{t+h-2}$$

Where $\hat{Z}_{T+h}$ is the forecast for the combined vector at horizon h_1, $\hat{A}_1$ and $\hat{A}_2$ are the estimated coefficient matrices from the VAR(2) model.

Extracting the Target Forecasts

The forecasts for our three target variables ($\hat{Y}_{T+h}$) are simply the **first three elements** of the forecasted combined vector $\hat{Z}_{t+h}$:

$$\hat{Y}_{t+h} = \text{Elements 1 to 3 of } \hat{Z}_{t+h}$$

This elegant extraction method ensures that the target forecasts are **dynamically consistent** with the factor forecasts and with each other.

OOS Performance Summary: DFM vs FAVAR

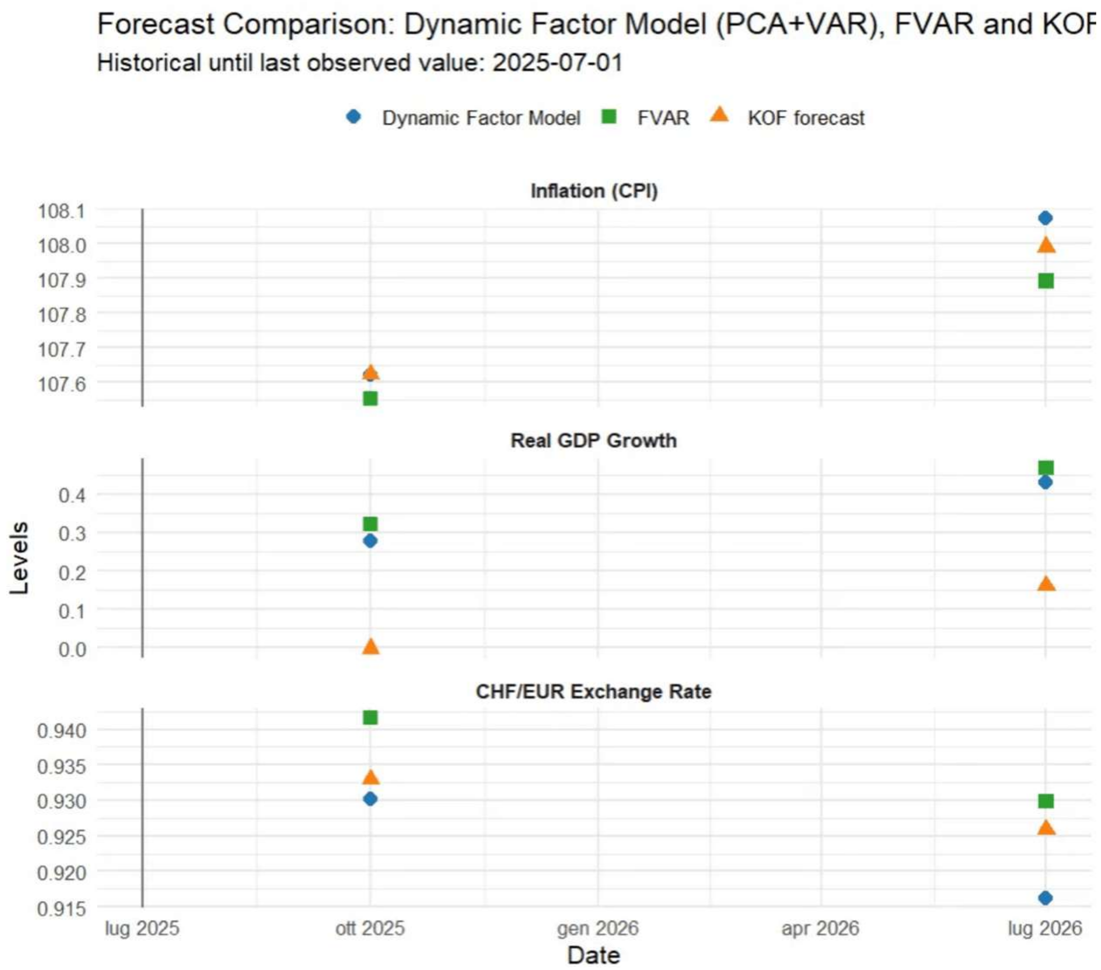
Variable	Horizon	DFM (RMSE)	FAVAR (RMSE)	Best Model
Real GDP Growth				
	h=1 (1Q)	0.613	**0.578**	**FAVAR**
	h=4 (1Y)	**0.557**	0.577	**DFM**
Inflation (CPI)				
	h=1 (1Q)	**0.524**	0.573	**DFM**
	h=4 (1Y)	**0.294**	0.352	**DFM**
Exchange Rate				
	h=1 (1Q)	**0.233**	0.395	**DFM**
	h=4 (1Y)	**0.135**	0.153	**DFM**

Key Insights

- Some high-frequency macro indicators contain short-term information relevant for GDP
- Many observed variables in FAVAR with limited sample size can introduce noise, reducing predictive power
- Latent factors in DFM provide a cleaner signal, extracting only the systematic co-movement useful for forecasting.

FAVAR Nowcasting Results: DFM vs FAVAR vs KOF

Variable	DFM	FAVAR	KOF
Real GDP Growth – Q4 2025			
	0.28%	**0.32%**	0.01%
Real GDP Growth – Q3 2026			
	0.43%	**0.47%**	0.18%
Inflation (CPI) – Q4 2025			
	107.62	**107.55**	107.62
Inflation (CPI) – Q3 2026			
	108.07	**107.89**	108.05
Exchange Rate (CHF/EUR) – Q4 2025			
	0.930	**0.942**	0.929
Exchange Rate (CHF/EUR) – Q3 2026			
	0.916	**0.941**	0.925



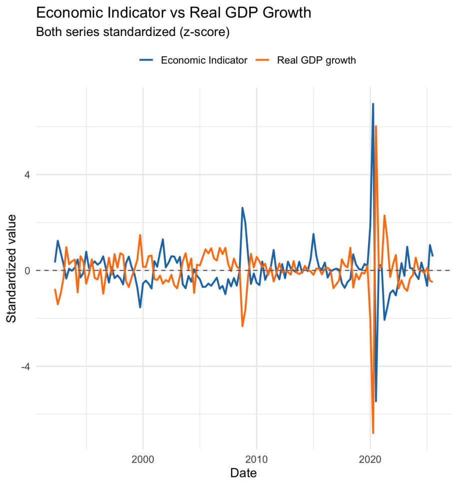
Economic Indicator

PC1 as an Economic Indicator

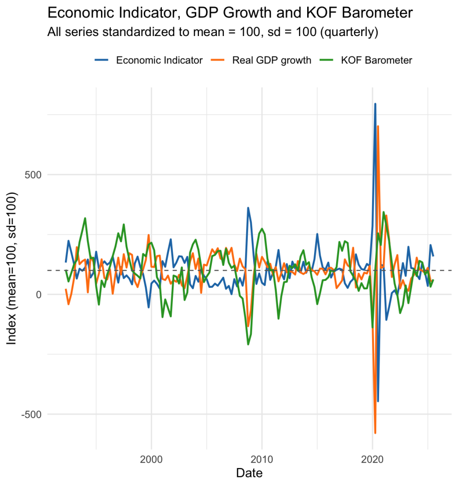
- **PC1**: first principal component from the standardized macro panel.
- Captures the dominant co-movement across indicators.
- **Sign note**: PCA signs are arbitrary; here PC1 moves *opposite* to real GDP and KOF.
- We plot all series standardized for comparability.

Reading: When PC1 rises, real activity/barometer tends to fall (and vice versa).

Economic Indicator vs. Real GDP (standardized)



Economic Indicator vs. KOF Barometer (standardized)



Conclusion

- **Data processing**:
  - Unit-aware transforms $\rightarrow$ stationarity; standardization; train/OOS split.
- **PCA**:
  - Reduced $N = 94$ to a small set of factors (r via Bai–Ng / elbow).
- **Forecasting (VAR / FaVAR)**:
  - Regressions and FaVAR on factors (and select originals) for $h = 1, 4$.
- **Economic indicator**:
  - PC1 used as a headline co-movement indicator; compared with GDP and KOF.

Next Steps: Tuning, Data, and Forecast Combination

- **Tuning (FaVAR / regressions)**
  - Select VAR lags p via **AIC/BIC** (baseline), confirm with OOS RMSE at $h \in \{1, 4\}$.
  - Choose factor count r by **forecast loss** (grid r), cross-check Bai–Ng ICs.
- **Additional data (timelier signals)**
  - Mixed-frequency: add **monthly** (IP, labor, prices) and **weekly/daily** (rates, spreads).
  - **Sentiment/News/Google Trends**, plus granular **KOF** survey balances.
- **Forecast combination**
  - Combine **FaVAR**, our PCA + VAR, and other models
  - Schemes: **equal weights**, **OLS stacking**, or **past-RMSE weights**.
  - Evaluate by rolling OOS RMSE/MAE; keep the best combo per target/horizon.

Citations

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